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ON AN EXACT SOLUTION OF THE EQUATIONS OF RELATIVISTIC GAS DYNAMICS*

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A particular solution of the equations of relativistic gas dynamics is obtained for one-dimensional adiabatic motions with plane symmetry. The solution is then used to construct an exact discontinuous solution describing a nonsteady motion of gas in the presence of shock waves.

A solution obtained by Sedov /1/ for the gas dynamic equations was coupled in /2,3/, in the case of one-dimensional, nonself-similar motions

$$u = \mp\mu\,[A + B\mu^{\nu(\gamma-1)}]^{1/2},\quad \rho = \frac{\mu^{\nu-1}}{r}\varphi'(\mu r),\ p = \mu^{\gamma\nu}\left[C + \frac{\nu(\gamma-1)}{2}B\varphi(\mu r)\right]$$

to a shock wave propagating through a gas at rest. Here the density ρ and pressure p are expressed in terms of an arbitrary function $\varphi(\mu r)$; A, B and C are arbitrary constants and γ is the ratio of specific heats. The velocity u, at any instant of time t, is a linear function of the distance r from the plane, axis or center of symmetry ($\nu = 1, 2$ and 3, respectively). Until then, exact discontinuous solutions were obtained only for single, isolated cases.

The relativistic investigation of the processes of propagation of simple waves through a two-dimensional space-time, and discussion of the strong discontinuities of the shock wave-type were carried out in /4,5/.

In the present case the equations of relativistic gas dynamics have the form

$$\frac{1}{\theta^2}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \frac{c^2}{p+\varepsilon}\left(\frac{\partial p}{\partial x} + \frac{u}{c^2}\frac{\partial p}{\partial t}\right) = 0,\quad \frac{\partial \ln V}{\partial t} + u\frac{\partial \ln V}{\partial x} - \frac{1}{\theta^2}\left(\frac{\partial u}{\partial x} + \frac{u}{c^2}\frac{\partial u}{\partial t}\right) = 0 \tag{1}$$

$$\frac{\partial (pV^\gamma)}{\partial t} + u\frac{\partial (pV^\gamma)}{\partial x} = 0;\quad \varepsilon = \frac{c^2}{V} + \frac{p}{\gamma-1},\quad \theta = \sqrt{1 - \frac{u^2}{c^2}}$$

Here ε is the "inherent" density of energy (including the internal energy) of the gas, and V is the "inherent" specific volume.

It can be confirmed that the system (1) has the following particular solution ($F(\xi)$ is an arbitrary function of the argument ξ and C_1 is an arbitrary constant):

$$u = \frac{c^2 t}{C_1 + x},\quad p = F(\xi),\quad V = c^2\left[\frac{\gamma}{1-\gamma}F(\xi) - \xi F'(\xi)\right]^{-1},\quad \xi = C_1^{-1}[(C_1 + x)^2 - c^2t^2]^{1/2} \tag{2}$$

We shall use the solution obtained to construct an exact discontinuous solution by coupling the solution (2) describing the motion of gas behind the shock wave, with the shock wave itself. The conditions at the shock wave can be written in the coordinate system in which the shock surface is at rest, in the form /6/

$$\frac{v_{1,2}^2}{c^2} = \frac{p_2 - p_1}{\varepsilon_2 - \varepsilon_1}\,\frac{p_{1,2} + \varepsilon_{2,1}}{p_{2,1} + \varepsilon_{1,2}},\quad \left(\frac{V_2}{V_1}\right)^2 = \frac{\varepsilon_1 + p_2}{\varepsilon_2 + p_1}\,\frac{\varepsilon_1 + p_1}{\varepsilon_2 + p_2} \tag{3}$$

where v_1 and v_2 are the rates of motion of gas before and behind the shock wave.

In a laboratory system where the shock wave front moves with velocity $\omega = dx_2/dt$, the velocity transformation formulas have the form

$$v_{1,2} = (u_{1,2} - \omega)(1 - \omega u_{1,2}/c^2)^{-1} \tag{4}$$

We shall assume that the shock wave propagates through the gas at rest without back pressure ($u_1 = 0,\ p_1 = 0$), and the initial specific volume is $V_1 = V_1(x)$. In this case we transform the equations (3) to obtain the following differential equation for determining the law of motion of the shock wave:

$$v_1[c^2(\gamma-1) + v_2(v_2 - \gamma v_1)]^2 = v_2(c^2 - v_1^2)[c^2(\gamma^2-1) + v_2(v_2 - \gamma^2 v_1)] \tag{5}$$

Here $v_1 = -dx_2/dt$ and the velocity v_2 is connected with $u_2 = c^2t/(C_1 + x_2)$ by the formula (4).

The simplest solution of (5) is $x_2 = ct + x_0$ where x_0 is the initial coordinate of the shock wave. Then, from (2) and (3) we finally obtain (C_2 is a constant of integration)

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$$\varepsilon_1 = C_2\left[\frac{\gamma^2}{2(\gamma-1)}-2\right]\left[\frac{1}{C_1^2}(C_1+x_0)(C_1-x_0+2x)\right]^{-\lambda}, \quad \varepsilon_2 = C_2\left[\frac{\gamma^2}{2(\gamma-1)}-1\right]\left[\frac{1}{C_1^2}(C_1+x_0)(C_1+x_0+2ct)\right]^{-\lambda}$$

$$p_2 = C_2\left[\frac{1}{C_1^2}(C_1+x_0)(C_1+x_0+2ct)\right]^{-\lambda}, \quad V_1 = \frac{c^2}{\varepsilon_1}, \quad V_2 = \left(\frac{\varepsilon_2}{c^2}-\frac{p}{\gamma-1}\right)^{-1}, \quad u_2 = \frac{c^2t}{ct+C_1+x_0}, \ \lambda = \gamma^2/[4(\gamma-1)]$$

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